Statistical aspects of some fatigue crack growth data

W.F. Wu a,*, C.C. Ni b

a Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan
b Department of Mechanical Engineering, China Institute of Technology, Taipei 11522, Taiwan

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Abstract

In this study, fatigue crack growth tests were performed for a considerable amount of specimens made of 2024T-351 aluminum alloy. In total, there are three data sets obtained, two of them are tested under constant-amplitude loading and the other under random loading. The results are reported in the present paper with special emphasis on the scatter and statistical aspect of these fatigue crack growth data. They are also compared with those of other available data sets. It is confirmed that the inhomogeneity of material does cause the scatter of fatigue crack growth. Whether the randomness of the applied load increases or decreases the scatter, however, needs to be investigated further since there is no unique result for all data sets compared. Crack exceedance probabilities of random crack size and cumulative distributions of random loading cycle obtained by solving probabilistic models of fatigue crack growth curves are also presented, and conclusions are drawn. Comments on chi-square goodness-of-fit test and equivalent loading investigation are made as well. © 2006 Elsevier Ltd. All rights reserved.

Keywords: Fatigue crack growth; Statistical analysis; Probabilistic model; Chi-square test; Equivalent load

1. Introduction

It is known that fatigue crack growth of engineering materials exhibits a wide range of scatter, even under the same loading condition in a strictly controlled environment with specimens cut from the same sheet. To take into account the scatter, it is important to analyze fatigue crack growth data from a statistical viewpoint. In fact, many researchers have carried out studies in this aspect [1–9], and many stochastic fatigue crack growth models have been proposed to depict the scattering of the crack growth process [10–20]. To proceed with the investigation, experimental data are always needed. However, it is rather time-consuming to carry out experiments in order to obtain sets of statistically meaningful fatigue crack growth data. Up to now, there are only a few data sets available in the literature. Among them, the most famous data set and frequently quoted by researchers [21,22] is perhaps the one obtained by Virkler et al. [1]. Some more frequently mentioned data sets include one reported by Ghonem and Dore [23] and the others released by Yang and his coworkers [24–28]. As a result, many stochastic fatigue crack growth models are either lacking experimental verification...
or verified by only one data set. It is obvious that more experimental data sets are needed to provide for researchers with tools to verify the applicability of their models. Therefore, in the present paper, three newly obtained and statistically meaningful fatigue crack growth data sets are presented, two of them under constant-amplitude loading test and the other under random loading test. Statistical analyses are performed on these data and their results are compared with those obtained from other published data sets. These newly obtained data are also used to verify several stochastic fatigue crack growth models proposed either by the writers or other researchers. Comments on results of the analyses are made at the end of the paper.

2. Fatigue crack growth experiment

2.1. Experimental setup and specimen

The experimental setup for obtaining the fatigue crack growth data consists of an MTS dynamic testing machine, a machine controller, a LabView signal generating/data acquisition system and a crack length measurement system. Compact tension (CT) specimens cut from a 2024-T351 aluminum-alloy plate were used for the fatigue crack growth tests. The dimensions of the specimens shown in Fig. 1 are 50.0 mm wide (counting from the loading line to the back face of the specimens) and 12.0 mm thick. The chemical compositions and mechanical properties of the material are tabulated in Tables 1 and 2, respectively.

Fig. 1. Geometry of specimen.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Chemical compositions of 2024-T351 (in wt.%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>Fe</td>
</tr>
<tr>
<td>0.07</td>
<td>0.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Mechanical properties of 2024-T351</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate strength (MPa)</td>
<td>Yield strength (MPa)</td>
</tr>
<tr>
<td>462</td>
<td>320</td>
</tr>
</tbody>
</table>
2.2. Constant-amplitude loading

For constant-amplitude loading fatigue experiment, two sets of specimen subjected to different loading conditions were tested. The first set, named CA1 (constant-amplitude loading set 1) hereafter for simplicity, consists of 30 specimens. They were tested under a sinusoidal load of \( p_{\text{peak}} = 4.5 \text{kN} \) and \( p_{\text{trough}} = 0.9 \text{kN} \), or in terms of stress ratio, \( R = p_{\text{trough}}/p_{\text{peak}} = 0.2 \), which results in stresses oscillating between 14.08 MPa and 70.41 MPa. The second set consists of 10 specimens and is named CA2 hereafter. They were tested under the sinusoidal load of \( p_{\text{peak}} = 6.118 \text{kN} \) and \( p_{\text{trough}} = 3.882 \text{kN} \), or \( R = p_{\text{trough}}/p_{\text{peak}} = 0.63 \), which results in stresses oscillating between 60.74 MPa and 95.72 MPa.

2.3. Random loading

For the random loading fatigue experiment, a set of 25 specimens named VA1 (variable-amplitude loading set 1) was tested. The random signals were generated in accordance with the spectral theory through the following expression [29–31]:

\[
g(t) = \sum_{k=1}^{n} \sqrt{2G(\omega_k)\Delta\omega} \cos(\omega_k t + \phi_k) \tag{1}
\]

in which \( g(t) \) is the random signals in time domain; \( G(\omega_k) \) is the power spectral density (PSD) function in frequency domain; \( \omega_k = \omega_L + (k - 0.5)\Delta\omega + \delta\omega_k \), \( \Delta\omega = (\omega_U - \omega_L)/n \) with \( \omega_L \) and \( \omega_U \) representing the lower and upper bounds of the PSD function, respectively, and \( n \) indicating the number of intervals divided between \( \omega_L \) and \( \omega_U \); \( \phi_k \) is a uniformly distributed random phase angle with values between 0 and \( 2\pi \), \( \delta\omega_k \) is another uniformly distributed random variable, added to avoid generating periodic signals, with values between \( -\Delta\omega' \) and \( \Delta\omega' \), and \( \Delta\omega' < \Delta\omega/10 \) assigned arbitrarily. In the present study, \( \omega_L, \omega_U, \) and \( n \) were chosen as 5 Hz, 15 Hz, and 200, respectively, in consideration of capacity of the MTS as well as accuracy of the generated signals. For these random signals, the degree of irregularity of the peaks and troughs can be judged by the irregular factor expressed as follows:

\[
\alpha = \sqrt{\frac{\int_{-\infty}^{\infty} \omega^2 G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega \int_{-\infty}^{\infty} \omega^4 G(\omega) d\omega}} \tag{2}
\]

For a stationary random process, if \( \alpha = 1 \), both the peak values and the trough values of the signals exhibit a Rayleigh distribution, which corresponds to a narrow-band random process. As the value of \( \alpha \) decreases, the irregularity of the random signals increases. It eventually reaches the limit case of a white-noise process with \( \alpha = 0 \). However, in practice, it has been pointed out that when the value of \( \alpha \) decreases from 1 to about 0.745, the signals are already very rough and close to a white noise [4]. In the present study, \( \alpha \) equals to 0.88 by considering a band-pass power spectral density having magnitude of 0.005 and an upper and a lower cutoff frequencies of 10 Hz and 5 Hz respectively.

To reduce the time lag between the MTS machine and the computer used for control purpose during the test, 80 blocks with 10,000 cycles in each block of random signals were generated and stored in a computer. The computer then transferred a block of signals at a time selected randomly among all blocks as inputs to the MTS machine. The process continued before a test was terminated. To view outcomes of the random loading, a small segment of signals are shown in Fig. 2. To express the signals in fatigue experimentalists’ terms, the random load calculated from all 80 blocks of signals has \( p_{\text{peak}} = 6.118 \text{kN} \), \( p_{\text{trough}} = 3.882 \text{kN} \), and \( p_{\text{amplitude}} = 1.118 \text{kN} \), all in the sense of mean value.

2.4. Experimental results

The experimental fatigue crack growth curves of CA1 under \( p_{\text{peak}} = 4.5 \text{kN} \) and \( R = 0.2 \), CA2 under \( p_{\text{peak}} = 6.118 \text{kN} \) and \( R = 0.63 \), and VA1 under variable-amplitude loading with mean \( p_{\text{peak}} = 6.118 \text{kN} \) and mean \( p_{\text{trough}} = 3.882 \text{kN} \) are shown in Fig. 3(a)–(c), respectively. A considerable degree of scatter including dispersion
of crack sizes at a specified loading cycle and dispersion of loading cycles at a specified crack size can be seen from these figures.

3. Statistical observation

3.1. Probability fit

Descriptive statistics was used to treat the experimentally obtained data. In particular, to investigate the distributions of loading cycles for crack size to reach specific values and distributions of crack sizes at specific loading cycles, both probability plot and chi-square goodness-of-fit test [32,33] were used. The data needed for analyzing the distributions of loading cycles and distributions of crack sizes were available from the raw data shown in Fig. 3, with a horizontal line and a vertical line intersecting the experimental curves shown in the figure, respectively. To be specific, numbers of loading cycles for the crack to grow to 51 selected loading cycles and lengths of crack at 51 selected loading cycles were recorded and analyzed. Probability distribution functions of normal, lognormal and Weibull and their corresponding papers were used to fit these data, and chi-square test was employed to examine the goodness-of-fits. The results are summarized in Table 3, in which numbers under each item from the third to the fifth columns indicate the amounts of best fitted sets among the selected 51 data sets mentioned above. All of them are in agreement with the indicated probability density function at a significance level of 5%. The numbers of data sets which do not pass the chi-square test are also shown in the sixth column of the same table. It was concluded that the random loading cycles and random crack sizes were best fitted by lognormal and Weibull probability distribution respectively for most studied cases. To visually illustrate some of these results, a few probability plots of random loading cycle to reach specific crack sizes for data set CA1 are shown in Fig. 4.

3.2. Variation of statistics

Based on the experimental results and applying descriptive statistics, the mean values, standard deviations, and coefficients of variation (COV) of random loading cycles for crack size to reach specific values and random crack sizes at specific loading cycles were also obtained and some of the results are shown in Fig. 5. In particular, the statistics of random loading cycles for crack size to reach a few specific values are tabulated in Table 4. It is interesting to note that the standard deviation of the loading cycles (to reach specific crack sizes)
Fig. 3. Experimental crack growth curves of data set (a) CA1, (b) CA2, and (c) VA1.
increases as the crack grows. The same trend is found for the standard deviation of crack sizes (at specific load-
ing cycles). However, the COV of loading cycle decreases smoothly as the crack size grows, while the COV of crack size increases rapidly as the loading cycle increases. It is noted that the curve labeled as CA3 in Fig. 5 was obtained from another available data set that will be discussed furthermore in the following paragraph.

To compare the above results with those obtained by other researchers, Table 5 was constructed in which CA3 indicates the data set published by Virkler et al. [1], and CA4 and VA2 are data sets published by Dominguez et al. [4]. Each of these data sets contains a considerable amount of crack growth curves which are considered statistically meaningful, and all specimens of these data sets are made of 2024-T3 series of aluminum alloy. Although the geometry, dimension, loading condition, and initial/final crack length recorded are somewhat different, it is still of interest to make comparison among these data sets. From the lower left plot of Fig. 5, it can be seen that the COV of loading cycle of data set CA3 reflects the same decreasing trend as those of CA1, CA2 and VA1 except at a few points. According to Dominguez et al. [4], the scatter of fatigue crack growth may be attributed to material inhomogeneity or random loading. Through examining COVs of loading cycle of data sets CA1, CA2, CA3 and CA4 in Table 5, it is concluded that the inhomogeneity of material does cause the scatter of fatigue crack growth, and the scatter could be considerably large. By comparing the

<table>
<thead>
<tr>
<th>Data set</th>
<th>Random quantity</th>
<th>Normal</th>
<th>Lognormal</th>
<th>Weibull</th>
<th>None</th>
<th>Total</th>
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<tbody>
<tr>
<td>CA1</td>
<td>Loading cycles</td>
<td>2</td>
<td>45</td>
<td>2</td>
<td>2</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Crack sizes</td>
<td>12</td>
<td>9</td>
<td>29</td>
<td>1</td>
<td>51</td>
</tr>
<tr>
<td>VA1</td>
<td>Loading cycles</td>
<td>4</td>
<td>39</td>
<td>3</td>
<td>5</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Crack sizes</td>
<td>15</td>
<td>8</td>
<td>24</td>
<td>4</td>
<td>51</td>
</tr>
</tbody>
</table>

![Fig. 4. Lognormal probability plots of random loading cycle for data set CA1.](image)
COV of CA4 under constant-amplitude loading, which is attributed to material inhomogeneity, and COV of VA2 under random loading, which is attributed to both material inhomogeneity and random loading, Dominguez et al. [4] pointed out that random loading does produce additional scatter other than material inhomogeneity. On the contrary, the present data sets CA1, CA2, and VA1 show a reduction of scatter from constant-amplitude loading to random loading. It does agree with Mann’s suggestion that the sporadic overloads of a random loading spectrum induces local plasticity that smoothes out the influence of inhomogeneities in the material and causes stress redistributions in the fatigue regions, and thus results in a more uniform fatigue response [34]. In conclusion, the effect of random loading on the scatter of fatigue crack growth needs to be studied further. Finally, by comparing the COVs of loading cycles and their corresponding maximum loading values for CA1, CA2, and CA3 presented in the study, it is found that the COV decreases as the

**Table 4**

<table>
<thead>
<tr>
<th>Data set</th>
<th>Crack size (mm)</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA1</td>
<td>(\mu)</td>
<td>19,441</td>
<td>32,792</td>
<td>41,917</td>
<td>48,042</td>
<td>51,984</td>
<td>54,356</td>
</tr>
<tr>
<td></td>
<td>(\sigma)</td>
<td>4,129</td>
<td>6,198</td>
<td>7,863</td>
<td>8,832</td>
<td>9,503</td>
<td>9,860</td>
</tr>
<tr>
<td></td>
<td>COV</td>
<td>0.212</td>
<td>0.189</td>
<td>0.188</td>
<td>0.184</td>
<td>0.183</td>
<td>0.181</td>
</tr>
<tr>
<td>CA2</td>
<td>(\mu)</td>
<td>72,082</td>
<td>128,221</td>
<td>166,138</td>
<td>194,794</td>
<td>215,083</td>
<td>230,645</td>
</tr>
<tr>
<td></td>
<td>(\sigma)</td>
<td>9,473</td>
<td>15,170</td>
<td>17,130</td>
<td>19,163</td>
<td>20,792</td>
<td>21,143</td>
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<td></td>
<td>COV</td>
<td>0.131</td>
<td>0.118</td>
<td>0.103</td>
<td>0.098</td>
<td>0.097</td>
<td>0.092</td>
</tr>
<tr>
<td>VA1</td>
<td>(\mu)</td>
<td>38,255</td>
<td>66,186</td>
<td>87,573</td>
<td>104,562</td>
<td>117,702</td>
<td>127,296</td>
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<tr>
<td></td>
<td>(\sigma)</td>
<td>3,290</td>
<td>5,139</td>
<td>6,710</td>
<td>7,204</td>
<td>8,410</td>
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<td>COV</td>
<td>0.086</td>
<td>0.078</td>
<td>0.077</td>
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<td>0.071</td>
<td>0.069</td>
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<td>----------</td>
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<td>--------------</td>
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<td>---------</td>
<td>--------------</td>
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<td></td>
</tr>
<tr>
<td>Specimen</td>
<td>CT</td>
<td>CT</td>
<td>CT</td>
<td>CT</td>
<td>CT</td>
<td>CT</td>
<td></td>
</tr>
<tr>
<td># Tested</td>
<td>30</td>
<td>10</td>
<td>68</td>
<td>18</td>
<td>25</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Loading</td>
<td>$P_{\text{max}} = 4.5$ kN</td>
<td>$P_{\text{max}} = 6.1$ kN</td>
<td>$P_{\text{max}} = 23.36$ kN</td>
<td>$P_{\text{max}} = 4.5$ kN</td>
<td>$P_{\text{max}} = 5.00$ kN ±random (diff. hist.)</td>
<td>$P_{\text{max}} = 4.83$ kN ±random (diff. hist.)</td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>18.00 mm</td>
<td>18.00 mm</td>
<td>9.00 mm</td>
<td>17.5 mm</td>
<td>18.00 mm</td>
<td>15.03 mm</td>
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</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.052 mm</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0034</td>
<td></td>
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<tr>
<td>COV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_t$</td>
<td>Fracture</td>
<td>32.00 mm</td>
<td>49.80 mm</td>
<td>27.8 mm</td>
<td>32.00 mm</td>
<td>25.3 mm</td>
<td></td>
</tr>
<tr>
<td>$a$ considered below</td>
<td>30.00 mm</td>
<td>30.00 mm</td>
<td>36.20 mm</td>
<td>30.00 mm</td>
<td>30.00 mm</td>
<td>25.3 mm</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>54,356</td>
<td>230,645</td>
<td>234,573</td>
<td>56,985</td>
<td>127,296</td>
<td>159,777</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>9,860</td>
<td>21,144</td>
<td>10,191</td>
<td>2,288</td>
<td>8,822</td>
<td>15,272</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>0.181</td>
<td>0.0917</td>
<td>0.043</td>
<td>0.040</td>
<td>0.0693</td>
<td>0.096</td>
<td></td>
</tr>
</tbody>
</table>
maximum loading increases. The result is consistent with Schijve’s observation. According to Schijve, it indicates that the maximum load level of the loading has some controlling influence on the fatigue scatter [2].

3.3. Equivalent load

For fatigue experimentalists, the concept of ‘equivalent load’ is sometimes used in the fatigue life prediction of components when the components are subjected to variable-amplitude loading [35–37]. To investigate the applicability of this concept, the peak and trough of data set CA2 were designed to be the same as those mean values of peak and trough of data set VA1. By doing so, both data sets resulted in the same mean stress and stress amplitude for the tested specimens in an equivalent sense. Adopting the equivalent-load concept and based on a crack growth formulation such as Paris law, one should be able to obtain the same fatigue life for both data sets. However, from observing the experimental results shown in Table 5, it is seen that fatigue life of data set CA2, 230,645 cycles, is almost twice the mean fatigue life of data set VA1, 127,296 cycles. The result indicates that one has to be careful in applying the equivalent-load concept to predict the fatigue life of components. In fact, in addition to the above so-called ‘mean equivalent load’, there are other forms of equivalent load including root mean square (RMS) [35], root mean cube (RMC) [36] and root mean \( m \) (RMM) [37] loads, where \( m \) indicates the exponent in Paris law. Among them, the mean equivalent load, in general, possesses the largest value of loading amplitude in an equivalent sense. Consequently, it will result in the smallest value of fatigue life in the prediction. It also indicates that, in the present study, the discrepancy between the predicted life and the experimentally obtained mean fatigue life of data set VA1 would be even larger if any of RMS, RMC and RMM loads was applied. The results are shown in Table 6, where RMM was excluded since \( m \) has a value between 2 (RMS) and 3 (RMC) in the present study, which will result in an equivalent loading amplitude between those of RMS and RMC. In summary, it is concluded that the concept of equivalent load is not applicable to the experimental data presented herein. The inconsistent result may be attributed to the overload effect of random loads, the sequential effect of loading spectra, the interaction between random loads, etc., which all have influences on the fatigue crack growth rate.

4. Stochastic analysis

Since there are more crack growth curves as samples, the data sets of CA1 and VA1 have been used for the verification of several stochastic fatigue crack growth models [18,19]. It was found that Bogdanoff and Kozin’s Markov chain model [10], Yang’s randomized Paris–Erdogan model [14] and a polynomial model proposed by the writers [19] can all describe data set CA1 very well, and the Markov chain and polynomial models can depict data set VA1 most accurately. The crack exceedance probabilities, indicating crack exceeds any given length at some specific loading cycles, for data set CA1 analyzed by Yang’s randomized Paris–Erdogan model are shown in Fig. 6. And the cumulative distributions of loading cycle to reach a few specific crack sizes for data set VA1 by the polynomial model are shown in Fig. 7. After a detailed study, it is concluded that each of the above models may be the most appropriate one to depict some particular sets of data but not necessarily the others. Subjected to the length of paper, the detailed descriptions and mathematics derivations of the above models are not included in the present paper. However, it is felt that mention of the works would make the content of the paper more complete. For those who are interested in the detail of these analyses, please refer to the corresponding papers by the writers [18,19].
Fig. 6. Crack exceedance probability for data set CA1 by Yang's randomized Paris-Erdogan model.

Fig. 7. Cumulative distributions of loading cycle for data set VA1 by polynomial model.
5. Concluding remarks

Three newly obtained fatigue crack growth data sets of 2024-T351 aluminum-alloy specimens are presented in the present paper. Statistical analysis of the experimental data is performed and the results are compared with those obtained from other published data sets. The newly obtained data are also used for the verification of several probabilistic fatigue crack growth models. It is hoped that the experimental fatigue crack growth data released and analyzed in this and its associated papers would provide researchers sources and references to further investigate the scatter of fatigue crack growth, and to verify or modify their stochastic or probabilistic crack growth models.

With regard to the statistical nature of the fatigue crack growth, the following phenomena are observed and comments are made at the end of the paper.

1. The loading cycle and crack size are best fitted by lognormal distribution and Weibull distribution, respectively, for all three data sets presented in this study.
2. The standard deviations of both loading cycle and crack size increase as fatigue crack grows for all three data sets presented in this study. Similar trends are also observed in other data sets.
3. The coefficient of variation of loading cycle decreases while the coefficient of variation of crack size increases as fatigue crack grows for all three data sets obtained by us. The decreasing trend of coefficient of variation of the loading cycle is also observed in Virkler’s data set.
4. The coefficient of variation of loading cycle decreases as the maximum load increases for those constant-amplitude loading data sets presented in this study.
5. The inhomogeneity of material does cause the scatter of fatigue crack growth, and the scatter may be considerably large. It, of course, also deviates from one manufacturer to the others.
6. In addition to the material inhomogeneity, whether random load increases or decreases the scatter of the fatigue crack growth needs to be studied further.
7. The concept of equivalent load used in the fatigue life prediction of components is not applicable to data sets presented in this study.
8. The randomized Paris–Erdogan and polynomial stochastic fatigue crack growth models are considered more appropriate to describe the fatigue crack growth data presented in this paper.

Finally, it should be mentioned that the crack size range is small for un-notched fastener holes presented by Yang and his coworkers and hence the statistical scatter of fatigue loading cycles is much larger than that of the Virkler’s data as well as the current new data.

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